

التمرين 1: إيجاد التحويل، واختيار الكلي
 1. معطيات المواد: $E = 200 \text{ GPa}$
 2. $I = 10^{-8} \text{ m}^4$

التمرين 1

$$U = \int_0^l \frac{M^2}{2EI} dx \quad (1)$$

1. U ، x ، l

2. إيجاد التحويل

$$w = \frac{\partial U}{\partial P} = \frac{1}{EI} \int_0^l M \frac{\partial M}{\partial P} dx$$

$$\frac{\partial M}{\partial P} = -x \quad \leftarrow M = -Px \quad (2)$$

$$w = \frac{1}{EI} \int_0^l (-Px)(-x) dx = \frac{P}{EI} \int_0^l x^2 dx$$

$$= \frac{P}{EI} \left[\frac{x^3}{3} \right]_0^l = \frac{Pl^3}{3EI}$$

3. U ، l

$$U = \int_0^l \frac{(-Px)^2}{2EI} dx$$

$$U = \frac{-P^2 x^3}{6EI} \Big|_0^l = \frac{-P^2 l^3}{6EI} \quad | \quad I = \frac{\pi d^4}{64}$$

$$U = -\frac{64P^2 l^3}{6E \pi d^4} \quad (3)$$

$$U = 15,56 \text{ N.m} \quad (3)$$

6) التمرين 6

(u, v, w) : دالة على (x, y, z)

$$\sigma = \begin{bmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & 0 \end{bmatrix}$$

$\sigma_{xz} = -G \sigma_y ; \sigma_{yz} = G \sigma_x ;$

$\sigma_x = \sigma_y = \sigma_z = \sigma_{xy} = 0$

ا.ب

$\sigma_{xy} = G \tau_{xy} ; \sigma_{xz} = G \tau_{xz}$

$\sigma_{yz} = G \tau_{yz}$

$$\begin{cases} \tau_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \tau_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \tau_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{cases}$$

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$\epsilon_x = \frac{\partial u}{\partial x} = 0 \Rightarrow u = F_1(y, z)$ — 1

$\epsilon_y = \frac{\partial v}{\partial y} = 0 \Rightarrow v = F_2(x, z)$ — 2

$\epsilon_z = \frac{\partial w}{\partial z} = 0 \Rightarrow w = F_3(x, y)$ — 3

$\epsilon_z = \frac{\partial w}{\partial z} = 0 \Rightarrow \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0$

$\sigma_{xy} = 0 \Rightarrow \tau_{xy} = 0 \Rightarrow \frac{\partial F_1(y, z)}{\partial y} = - \frac{\partial F_2(x, z)}{\partial x}$

$\frac{\partial F_1(y, z)}{\partial y} = G_1(z) ; \frac{\partial F_2(x, z)}{\partial x} = G_2(z)$
 $G_1(z) = -G_2(z) \Rightarrow \int G_1(z) dz = - \int G_2(z) dz + C$

$$F_1(x, y, z) = G_1(z)y + H_1(z) \quad \text{--- (4)}$$

$$F_2(x, y, z) = -G_1(z)x + H_2(z) \quad \text{--- (5)}$$

من (1) و (2) و (3) نحصل

$$\frac{\partial F_2(x, y, z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial y} = 6x \quad \text{--- (6)}$$

$$\frac{\partial F_1(x, y, z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial x} = -6y \quad \text{--- (7)}$$

بمقارنة (4) و (5) و (6) و (7) نجد:

$$\textcircled{1} \quad -\frac{\partial G_1(z)}{\partial z}x + \frac{\partial H_2(z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial y} = 6x$$

$$\frac{\partial G_1(z)}{\partial z}y + \frac{\partial H_1(z)}{\partial z} + \frac{\partial F_3(x, y, z)}{\partial x} = -6y \quad \text{--- (8)}$$

$$\begin{cases} -G_1'(z)x + H_2'(z) - 6x = -\frac{\partial F_3(x, y, z)}{\partial y} \\ G_1'(z)y + H_1'(z) + 6y = -\frac{\partial F_3(x, y, z)}{\partial x} \end{cases} \quad \text{--- (9)}$$

من (9) نجد أن $G_1'(z)$ و $H_1'(z)$ و $H_2'(z)$ لا يعتمدان على x و y

$$G_1'(z) = A_1 \Rightarrow G_1(z) = A_1 z + B_1$$

$$H_1'(z) = A_2 \Rightarrow H_1(z) = A_2 z + B_2$$

$$H_2'(z) = A_3 \Rightarrow H_2(z) = A_3 z + B_3 \quad \text{--- (10)}$$

$$\begin{cases} -A_1x + A_3 - 6x = -\frac{\partial F_3(x, y, z)}{\partial y} \\ A_1y + A_2 + 6y = -\frac{\partial F_3(x, y, z)}{\partial x} \end{cases}$$

$$\begin{cases} -A_1 xy + A_3 y - 2xy = -F_3(x, y) \\ A_1 yx + A_2 x + 6xy = F_3(x, y) \end{cases}$$

$$F_3(x, y) = -\frac{1}{2} (A_2 x + A_3 y) + H_3(y)$$

$$\begin{cases} \frac{\partial F_2(x, z)}{\partial z} - \frac{A_3}{2} = 6x \\ \frac{\partial F_1(y, z)}{\partial z} - \frac{A_2}{2} = -6y \end{cases} \Rightarrow$$

$$\begin{cases} F_2(x, z) = \frac{1}{2} A_3 z + 6xz + H_3(z) \\ F_1(y, z) = \frac{1}{2} A_2 z - 6yz + H_3(z) \end{cases}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} = 0 \Rightarrow -H_3(z) = 0$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = 0 \Rightarrow H_4(z) = 0$$

$$\sigma_{xz} = -G \sigma_y = G \gamma_{xz} = G \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

$$\sigma_{xz} = 0$$

$$\Rightarrow -6y = \frac{1}{2} A_2 - 6y + \frac{1}{2} A_2 \Rightarrow A_2 = 0$$

$$\begin{cases} u = -6yz \quad \text{①} \\ v = 6xz \quad \text{①} \\ w = 0 \quad \text{①} \end{cases}$$

08 03 Ex 2-1
 que h. x

① $w = 0$
 ① $\frac{\partial w}{\partial n} = 0$

① $w = 0$; $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$
 ① δ

① $M = 0$, $w = 0$
 ① $\frac{\partial w}{\partial n} = 0$ $v_x = (q_n + \frac{\partial M}{\partial y}) = 0$
 ①